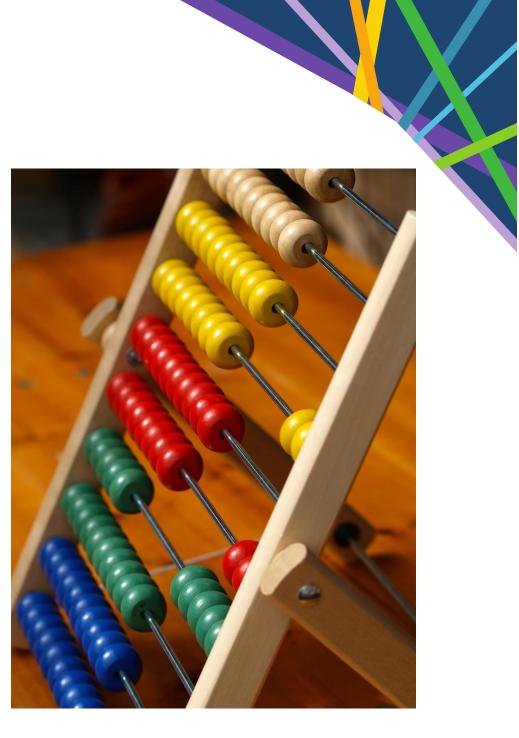


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## **Calculation Policy**

**Primary** 

Draft 1



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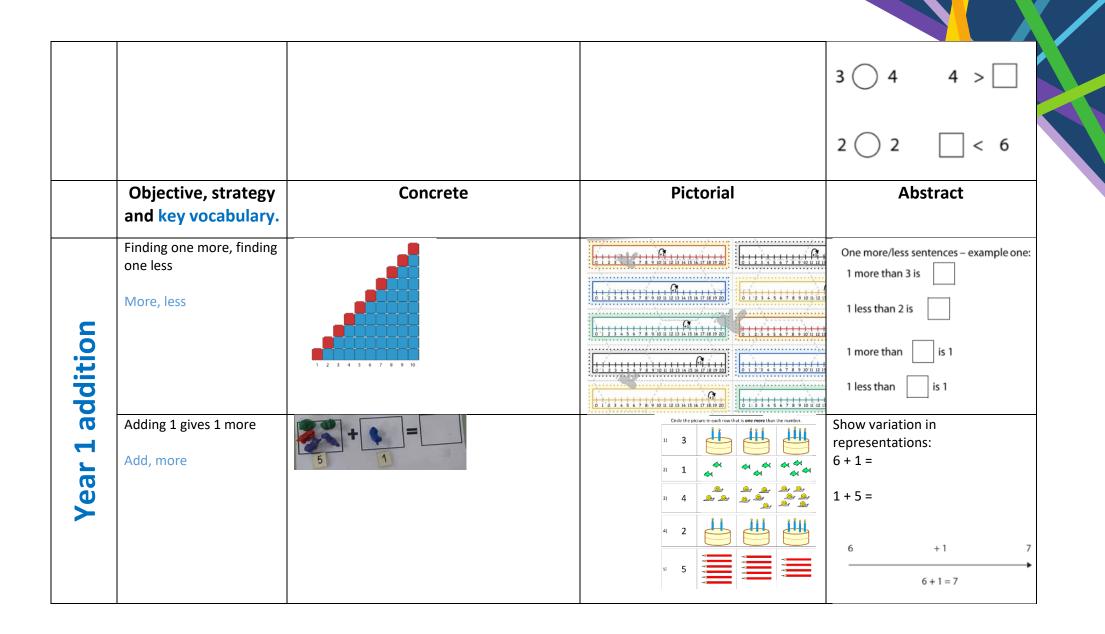
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Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
Comparing objects, groups of objects. Length, weight, mass, heavier, lighter, same, equal.	Compare children's height, compare distance, compare mass of objects. Compare multiple objects (use bears, jewels, cubes etc to create groups of different sizes to compare). Use of pan balance with numicon to show equivalence = < >		
Using < > and = Fewer, more, less than, more than, equal to, fewer than	Use multilink to create different amounts.		Use variation with missing boxes:



Augmentation— increasing an amount	Use FIRST, THEN, NOW and range of practical situations for showing augmentation. E.g. first there were three children on the carpet then 2 more came. Now there are 5 children on the carpet.	First Then N	$4 + 3 = 7$ $4 + ? = 7$ $\frac{4 + 3}{4 + 3} = 7$
Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
Stories of numbers within 10	Children work with tens frames (e.g. egg boxes of 10) and other practical apparatus to describe 'stories of ten'.	10+0     0000       9+1     0000       8+2     0000       7+3     0000	Calculations within 10, e.g: 3 + 4 = 7 7 + 0 = 7 6 + 1 = 7 2 + 5 = 7 1 + 6 = 7 0 + 7 = 7 5 + 2 = 7 4 + 3 = 7

V

Year 1 addition	Combining two parts to make a whole: part- whole model	<image/>	Image: state stat	4 + 3 = 7 $10 = 6 + 4$ $5$ $3$ Use the part-part whole diagram as shown above to move into the abstract.
	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract

Regrouping to make 10. (This skill will be essential when moving onto column addition later.)	6 + 5 = 11	Use pictures number line. or partition th number to m	
regroup	Start with the bigger number and use the smaller number to make 10.	9 + 5 = 14 1 4 +1 +4 +1 +4 +1	
Represent & use number bonds and related subtraction facts within 20 Number bonds	Start with the big- ger number and use the smaller number to make 10. Use ten frame	Use pictures or a number line. Regroup of partition the smaller number using the pa part whole model to make 10. 9 + 5 = 14	Emphasis should be on the language '1 more than 6 is equal to 7.' '5 more than 9 is equal to 14' '2 more than 5 is equal to 7'
		$\begin{array}{c c} \hline \\ \hline $	

Addin	g I and 2		Bonds to	0 10	A	dding 10		Bridg	ing/		
	5 - 4110 2					33115		compen	-		
Do	oubles		Adding	g 0	Nea	ar double	s				
+	0	Ì,	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
Ι	I + 0	1+1	1 + 2	1 + 3	1 + 4	1 + 5	I + 6	1 + 7	1 + 8	+ 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
		ion and subtraction: tens boundary		
Year 2	Adding multiples of ten	Model calculation using dienes or beadstrings. 20 + 30 = 50	Draw representations for dienes to represent tens and ones, eg: + + + + + + + + + + + + + + + + + + +	20 + 30 = 50 30 + 20 = 50 ? + 20 = 50 50 = ? + 20
Ye	Recall and use known number facts.	Children use the 'cherry' part-part- whole model to create concrete representations of 20 in different ways.	20 20 12 7	20 

	Objective, strategy and key	Concrete	Pictorial	Abstract
n	vocabulary. Use related number facts.	Ted Sam	$\begin{array}{c} \vdots & + & \vdots & = & \vdots \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\$	Show that: 3 + 4 = 7 30 + 40 = 70 300 + 400 = 700
- 2 addition	Use bar models.	Concrete to pictorial - drawing	Show that 12 + 8 = 20 using real-life objects, then cubes, then drawing the objects/cubes onto a bar model template.	Progress to using digits in the bar model template (showing proportion): 20 8 12
Year	Add a 2 digit number and ones.	Use tens frames to represent 'bridging' the ten with the ones. Use 'real life' egg box tens frames with concrete manipulatives.	Use a number line: 35 + 3 = 35 + 3 =	Explore related facts: 35 + 3 = ? ? + 3 = 38 38 = 35 + ? 38 3 35

	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
	Add a 2 digit number and tens.	Use dienes or numicon:	50 + 2.0 = $+ 10 + 10$ $50 = 60 = 70$	Bar model – represent the commutative law:         21         10       11         10 + 11 = 21       21         21       10         11 + 10 = 21       10
Year 2	Add two 2 digit numbers.	Model using dienes (including exchanging ten ones for one ten): Dienes and part-part-whole model: 45 + 23 = 68 47 + 23 = 68 60 + 8 = 68 47 + 8 = 68 Leading to exchanging: T2	$\begin{array}{c} +20 +5 & 0r +20 +3 +2 \\ \hline 47 & 67 & 72 \end{array}$ Use number line and bridge ten using part whole if necessary.	Progress to expanded written method: 40 + 7 + 20 + 5 60 + 12 = 72 25 + 47 20 + 5 + 40 + 7 20 + 40 = 60 5 + 7 = 12 60 + 12 = 72

Y2 continued:		Pictorial: First Then	Then Now	(4) + 7 + (6) = 10 + 7	
Add three one digit numbers.				10 = 17	
		Use part part		Combine the two numbers that make/ bridge ten then add on the third.	
		whole to show			
	First, look to find number bonds to 10.	magic ten	2 + 3 + 8 10 + 3 = 13		

Ν

	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
	New vocab used for Y3 add inverse.	lition and subtraction: increase, column	addition, vertical, 'carry', expanded, compact, exchan	ge, decrease, column subtraction,
Year 3	Column addition (without regrouping)	Using a place value chart, model the tens and ones using dienes or numicon: 23 + 34          Image: Construction of the tens of tens of the tens of	Children progress to drawing the PV counters onto a place value chart.         Hundreds       Tens       Ones         Image: Im	3       5       6         +       2       2       1         5       7       7
	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract

Column addition (with Expanded method is modelled Children are able to draw representations of a regrouping). first: Units Tens Formal column addition place value counter or dienes, using hand drawn 200 + 40 + 7 39 method. circles or sticks/dots to represent amounts:  $\frac{100 + 20 + 5}{300 + 60 + 12} = 372$ Carry figures 15 4 5 247 **Expanded** method +<u>125</u> Model using dienes or numicon. 12 Progress to place value counters: Compact method 60 <u>300</u> 372 11 addition 1 1 11 Progressing to formal column 1 (compact) method: 1 1 536 11 m Year 85

	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
	New vocab introduced for	Y4-6 addition and subtraction: hundreds	boundary, thousands boundary, tens of thousands	s boundary, BIDMAS.
	Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.		+9254 4+8=12 3768
9		Doucards         Jundredi         Juns         ones           Image: Constraint of the second	7 1 5 1	Continue from previous work to carry ones, tens and hundreds. Relate to measure and money.
4 -			Draw representations using pv grid.	
Years	<ul> <li>Y5—add numbers with more than 4 digits.</li> <li>Add decimals with 2 decimal places, including money</li> <li>Y6—add several numbers of increasing complexity</li> </ul>	ones     tenths     hundredths       1     01     01     00     00       1     01     01     00     00       1     01     01     00     00       1     01     01     00     00       1     01     01     00     00       1     01     01     00     00       1     01     01     00     00	2.37 + 81.79 <u>tens</u> on as <u>tents</u> <u>hundredities</u> 00 000 0000 00000 00 0000 0 00000 00 0000 0 00000	Y6: $ \begin{array}{r} 22,634 \\ + 15,673 \\ \underline{38,207} \\ 1 1 \\ f 127.67 \\ + f 38.45 \\ \underline{f 166\cdot12} \\ 1 1 1 \\ 89,472 \\ 63,673 \\ + 3.016 \\ \hline \end{array} $
	Including adding money, measure and decimals with different numbers of decimal points.		6	$\begin{array}{c} + \underbrace{3,016}_{156,161} \\ 1 \underbrace{56,161}_{1111} \\ \\ \text{lnsert zeros for place holders.} \end{array} \begin{array}{c} 1.437 \\ 0.600 \\ + 3.020 \\ \hline 4.057 \\ 1 \end{array}$

		Sub	otraction	
	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
	Read, write and interpret calculations involving the '-'sign.	Physically remove items to view subtraction as 'taking away', such as groups of bears:	Cross out items represented pictorially:	Use the – sign in written calculations: 9 – 3 = 6 10 – 3 = 7
Year 1			Use a number line to count back:	Include the effect of subtracting zero.
	Represent and use number bonds and related subtraction facts within 20 Part-Part-Whole model	Link to addition with use of the part- whole model representation. 6 = 4+2 6-4 =2 Model using cubes and other manipulatives on large part-whole model templates		part part 5 2 7 whole
			Extend to use of pictorial representations.	Use numbers in the part-whole model; extend representation of part-whole 'cherry' model to various orientations.

	Compare numbers by finding the difference	Use counters to represent difference, e.g: The cars in the car park	సంపంపంపం సుపుపుపు సుపుపుపు శ్రీ శ్రీ శ్రీ శ్రీ శ్రీ శ్రీ శ్రీ శ్రీ	Introduce numbers to the concept of finding the difference, e.g. The difference between 10 and 6 is 4. The part that is the same is 6, the part that is different is 4 (demonstrated by the pictorial representation).
	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
ar 2	Subtract efficiently by making ten.	Make 15 on the $15 - 9 =$ ten frame. Take 5 away to make ten, then take 4 more away so that you have taken 9. 15 - 9 = 10 15 - 5 = 10 10 - 4 = 6 15 - 9 = 6	15 - 9 = $-4$ $-5$ $-5$ $-5$ $-5$ $-5$ $-5$ $-5$ $-5$	$16 - 9 =$ How many do we take off first to get to 10? How many left to take off? $10 \frac{16}{9}$ $11$ $2 6$
Year	Counting on to next ten (Progression - crossing one ten, crossing more than one ten, crossing the hundreds.)	Use beadstrings to model taking ten, then the remaining amount. E.g. 37 - 12	e.g. 37 – 12 = 25 25 27 37 -2 -10 NB: The second number has been partitioned and subtracted separately in tens and ones.	<ul> <li>37 - 12 = ?</li> <li>37 - ? = 25</li> <li>25 = 37 - ?</li> <li>Children are encouraged to use the inverse to recognise the relationship between addition and subtraction.</li> </ul>

	Subtracting a multiple of 10 Objective, strategy and key vocabulary.	Dienes are used to model physically taking away ten.	Children draw representation of dienes (and cross out the correct amount to take away):	32 - 10 = ? $22 = ? - 10$ $32 - 20 =$ $32 - 30 =$ Abstract
	Subtract a single digit from a two digit number (No regrouping)	Explore that 9 - 3 = 6 so 29 - 3 = 26 etc	$\begin{array}{c} -3 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array}  9 - 3 = 6$	9 - 3 = 6 19 - 6 = 13 29 - 6 = 23 and so on
Year 2	Partitioning two digit numbers to subtract without regrouping.	Tens  Ones	Children draw pictorial representations of this, crossing out the tens and ones:	70 5 -40 2 30 3

Regroup a ten into ten ones.	Think: I hove 3 lens and v ones. I want to take and y ones. I want to take and	Children draw pictorial representations of this:	<ul> <li>42 - 27 = ?</li> <li>27 + ? =</li> <li>42 - ? = 27</li> <li>The inverse relationship between addition and subtraction continues to be reinforced.</li> </ul>
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	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
Year 3	Column subtraction without regrouping	Use dienes or numicon to model: 34-13=21 1000	Draw representations to support independent understanding:	Use extra 'expanded' step if necessary to support $90 + 8$ understanding: -30 + 6 60 + 2 56 - 12 -12 53
	Column subtraction with regrouping	Use dienes to model exchanging one ten for ten ones, for example in the calculation: 56 – 29	Draw representations to support:	Use formal written representations. <sup>6</sup> <sup>×</sup> 12 <u>56</u> <u>24</u> Move to three digit subtraction:

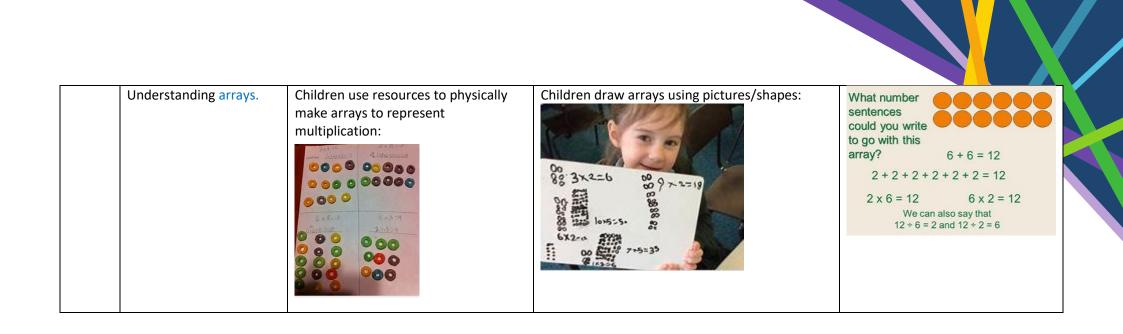
		<ul> <li>8948 (The digits in the right hand column are subtracted first and children continue to work from right to left.)</li> </ul>	
		685	

	Objective, strategy	Concrete	Pictorial	Abstract
	and key vocabulary.			
Years 4 - 6	Subtracting tens and ones. Year 4 subtract with up to 4 digits. Introduce decimal subtraction through context of money.	Place value counters are used to secure understanding:	Children draw representations of these (or dienes) to exchange/'cross out' on a place value chart.	5 5 2 2 2 2 2 2 2 4 8 8 8 8 8 8 8 8 8 8 8 8 8
	Year 5- Subtract with at least 4 digits, including money and measures	Children can continue to use PV counters/dienes to support calculations.	As year 4.	$\begin{array}{r} 7 & 9 & 9 & 1 \\ \hline 8 & 0 & 0 & - \\ \hline 6 & 7 & 3 \\ \hline 7 & 3 & 2 & 7 \\ \hline \\ \hline \\ Children are \\ encouraged to fill any \\ empty decimal places \\ with zero to show the \\ place value in each \\ column. (See right.) \\ \hline \\ \hline \\ \hline \\ 4 & 3 & \cdot 8 & 8 \end{array}$

Year 6—Subtract with increasingly large and	As above.	As above.	Υ X X , 6 9 9 - <u>8 9, 9 4 9</u> - 6 0, 7 5 0
more complex numbers and decimal values.			1/ 10 '5 · 3/4 '1 9 kg
			$- 36 \cdot 080 kg \\ 69 \cdot 339 kg$

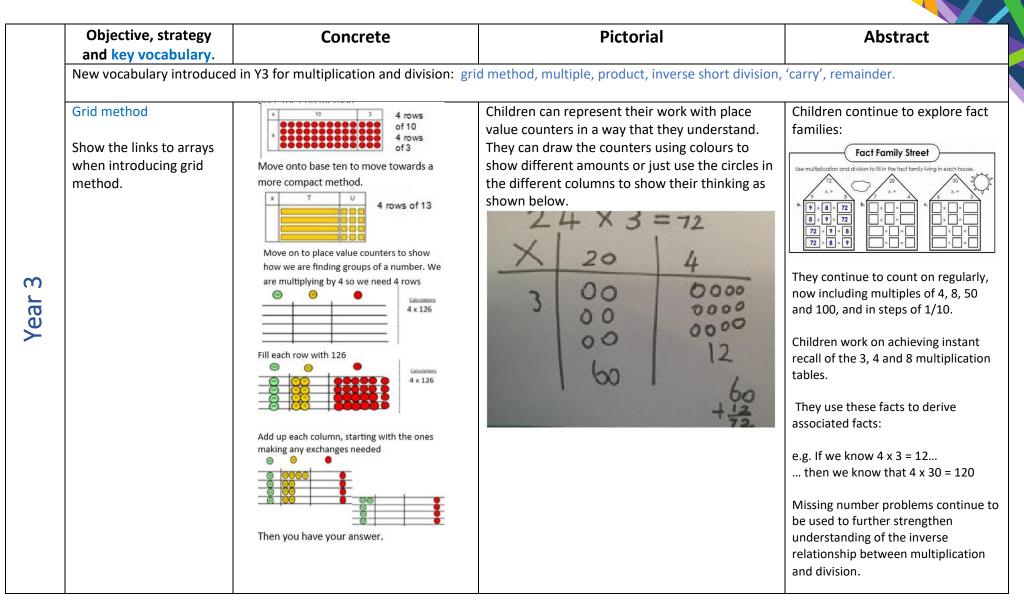
		Mult	ciplication	
	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
	Key vocabulary used in Y1	multiplication and division: share, share I	petween, share equally, group, groups of, lots of, a	rray.
Year 1	Doubling.	Use numicon, cubes and other manipulatives to demonstrate the concept of doubling: $+ = = \infty$ $+ = = \infty$ $+ = = \infty$	Use pictures as a method of calculating and representing doubles: 3+3=6	Use numbers and symbols, extending to 'one more than double': Notifie

	Counting in multiples.	Children 'skip count' as teacher models this using a beadstring or groups of cubes/objects.	Children use/make pictorial representations. They can create these themselves and teachers can provide images to support understanding.	Children count aloud in multiples (teacher can use counting stick/hoop to support). Children can write number sequences in multiples: 2,4,6,8 5,10,15,20
	Make equal groups and count the total.	Use concrete resources to make equal groups.	Use representations to show equal groups: Children solve problems using pictures, e.g. Joe thinks that 2 x 2 = 5. Use to prove your answer and say whether you think Joe is right or wrong.	$2 \times 5 = 10$ $10 = 5 \times 2$ Double 5 is 10 $2 \times ? = 8$
	Objective, strategy and key	Concrete	Pictorial	Abstract
Year 1	vocabulary. Repeated addition.	4+4+4=12 5 + 5 + 5	Use hops along a number line: 2 $2$ $2$ $2$ $2$ $2$ $2$ $2$ $2$ $2$	2 + 2 + 2 + 2 + 2 = 10 5 hops of 2 = 10



	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
Year 2	New vocab for Y2 multiplic	ation and division: divide, divided by, divi	ded into, division, left, left over.	
	Doubling two digit numbers.	Model doubling using dienes/PV counters/numicon; partition to reinforce understanding of place value.	Children draw representations of dienes to support calculations for doubling.	Partition a number, then double each part before recombining to total. 16 10 10 10 10 10 10 10 10
	Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. 5+5+5+5+5+5+5+5=40	Children use number lines, 'empty' number lines and bar models: +4 $+4$ $+4$ $+40 4 8 120 1 2 3 4 5 6 7 8 9 10 11 129 9 10 11 12$	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 $4 \times 3 =$

	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract	
Year 2 multiplication	Multiplication is commutative.	Make arrays using different concrete resources (e.g. counters, cubes): Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	Children create representations of arrays to support calculations and develop understanding of commutativity:	$7 \times \square = 14$ $14 = \square \times \square$ $\square \times 2 = 14$ $14 = 2 \times \square$ $\square \times \bigcirc = 14$ $14 = \square \times \square$	
Ye	Using the Inverse (This should be taught alongside division, so pupils learn how they work alongside each other.)	Use representations of arrays as above.	$ \begin{array}{c} 8\\ 4\\ 2\\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	5     40     8       Use only these mombers to make a different number sortence each time.       One is done for yow.       5 $\times$ 8     =     40 $\searrow$ $x$ $=$ $\bigcirc$ $\bigcirc$ $\times$ $=$ $\bigcirc$ $\div$ $=$ $\bigcirc$	



	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
ar 4	New vocab introduced in 4 fo	r multiplication and division: factor, quotient	, divisor.	
Ye	Recap grid method from Year 3 extending to multiply	See above (grid method Y3)		
	a 3 digit number by a 1 digit number.			

	Extend to: Column multiplication	Children use dienes/PV counters to support calculations, beginning with calculations requiring no regrouping:Image: constraint of the second s	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	Short multiplication method:123 x 5I 23 x 5I 23 x 5x 5
--	-------------------------------------	--	--	---

	Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.		10	8	123 × 45 1st step 123 × 45
			10	100	80	$\frac{2 \text{ A5}}{615} (123 \times 5)$ $\frac{2 \text{ nd step}}{123}$
			3	30	24	×     45       615     0       (because we are multiplying tens)
						$3rd step 123 \times 45615 (123 \times 5)4920 (123 \times 40)5535 (615 \pm 4920)$
						Children continue to count on regularly, including steps of powers of 10. (They multiply whole numbers by 10, 100 and
						1000, including decimals.) Understanding that the scaling of multiples of 10 can be used to convert between units of measure is explored (e.g. metres to kilometres means multiply by 1000.)
						Pupils use practical resources and jottings to explore equivalent statements: e.g. 4 x 35 = 2 x 2 x 35
9	Objective, strategy and key vocabulary.	Concrete		Pictor	ial	Abstract
Year	Continue to use methods for long division as Y5.	See Y5.				

Multiplying decimals up	Children are also introduced to multiplication
to 2 decimal places by a	of numbers with up to two decimal places by
single digit.	one-digit and two-digit numbers.
	They begin by starting with simple cases, such as: 0.4 x 2 = 0.8
	They then move on to more complex problems: e.g.
	3 · 1 9 × 8 2 5 · 5 2
	<ul> <li>Line up the decimal points in the question and the answer I Remember that the single digit belongs in the 'ones' column</li> </ul>
	NB: This method works well for multiplying money (£.p) and other measures.
	Children experiment with order of operations 'BIDMAS'.

			Division	
Year 1	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract

Division as sharing.		Children use pictures or shapes to share quanti- ties.	"10 shared between 2 is 5." "12 shared between 3 is 4."
	Thave 10 cubes, can you share them equally in 2 groups?	Sharing: A 12 shared between 3 is 4	

	-			
	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
	Division as sharing.	"I have 15 cubes. Can you share them equally between 3 friends?"	Children use pictures to support understanding of sharing: 15 sweets shared between 3 people 15 ÷ 3 = 5	12 ÷ 3 = 4
Year 2		15-3=5	Children use bar models to support conceptual understanding:	9 ÷ 3 = 3
			9 3 3 3	
	Division as grouping.	Divide objects into equal groups. How many groups of 3 can be made with 12 flowers?	Use number lines and bar models for grouping:	20 ÷ 5 = ?
			There are counters in each of the 5 groups.         A bar model can show how the parts of a problem are related.         • Complete the bar model to show 20 prizes divided into 5 equal groups.         20 prizes	

	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract
	Division as grouping.	PV counters/numicon can be used to demonstrate grouping: 24 divided into groups of 6 = 4 96 ÷ 3 = 32	Continue to use bar modelling, to represent division and to show the relationship between division and multiplication: 20 ? 20 $\div$ 5 = ?	96 ÷ 3 = 32 There are 96 cars. There are three car parks. How many cars can park in each car park? 3 2 $3 9 6$
Year 3	Division with arrays.	Use concrete resources to demonstrate, using arrays, the relationship between multiplication and division: Array Model Division and Multiplication $15 \div 3 = 5$ Total Number Number in of groups each group $3 \times 5 = 15$ Number Number in Total of groups each group	Draw an array and use lines to split the array into groups to make multiplication and division sentences : $Arroy for Pivision$ $12 - 4 = 3$ $- * * * * *$ $- * * * * *$ $- * * * * *$ $12 \div 3 = 4$ $3 \times 4 = 12$ $4 \times 3 = 12$	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Objective, strategy and key	Concrete	Pictorial	Abstract
Vocabulary. Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over:	Ask "How many 5s in 40?" 0 5 10 15 20 25 30 35 4 Example with remainder: 38 + 6	s with a remainder of 2

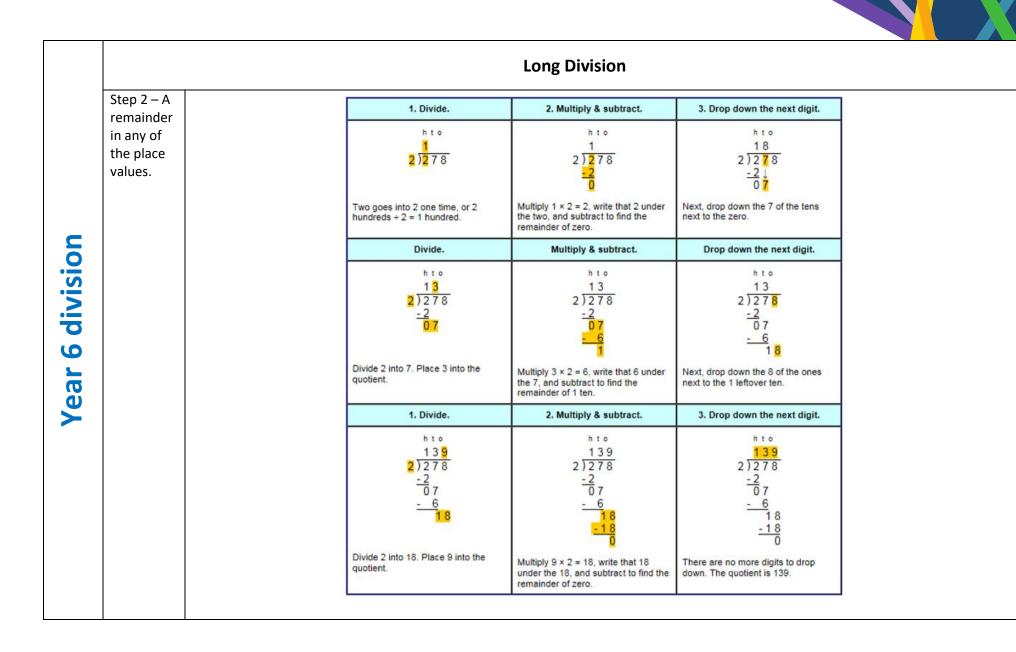
	Objective, strategy	Concrete	Pictorial	Abstract
	and key			
	vocabulary.			Begin with divisions that divide equally with
	Divide at least 3 digit		Children can replace PV counters/dienes for	no remainder.
	numbers by 1 digit.	1221	pictorial representations (which they may draw or may be provided for them).	2 1 8
	Short division.	3317		
		130/00		4 8 7 2
		100 10 10	3)568	Move onto divisions with a remainder.
			0 000	
9 -				<u>86</u> r2
4			thousands hundreds tens ones	5 4 3 2
LS				5 4 5 2
Years				Finally move into decimal places to divide the
$\succ$				total accurately.
			1 monthemi	14.6
				16 21
				3 5 5 1 1 . 0

Year 6	Year 6 Long Division - Use the formal long division method – either DMSB or chunking	

Step 1 – A remainder in the ones.	$\begin{array}{r} h \ t \ o \\ 0 \ 4 \ 1 \ R1 \\ 4 \ \end{array} \begin{array}{r} 1 \ 6 \ 5 \end{array}$ 4 does not go into 1 (hundred). So combine the 1 hundred with 4 goes into 16 four times. 4 goes into 5 once, leaving a remainder of 1. $\begin{array}{r} th \ h \ t \ o \\ 0 \ 4 \ 0 \ 0 \ R7 \\ 8 \ \end{array}$	with the 6 tens (160).
	8 does not go into 3 of the thousands. So combine the 3 the 8 goes into 32 four times (3,200 + 8 = 400) 8 goes into 0 zero times (tens). 8 goes into 7 zero times, and leaves a remainder of 7.	busands with the 2 hundreds (3,200). $ \begin{array}{r}             h \ t \ o \\             0 \ 6 \ 1 \\             4 \ \overline{) \ 2 \ 4 \ 7} \\             - 4 \\             3 \\             When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3. Check: 4 × 61 + 3 = 247             th h t o              0 \ 4 \ 0 \ 2} \\             4 \ \overline{) \ 1 \ 6 \ 9} \\             - 8 \\             1 \end{array} $ When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

## Long Division

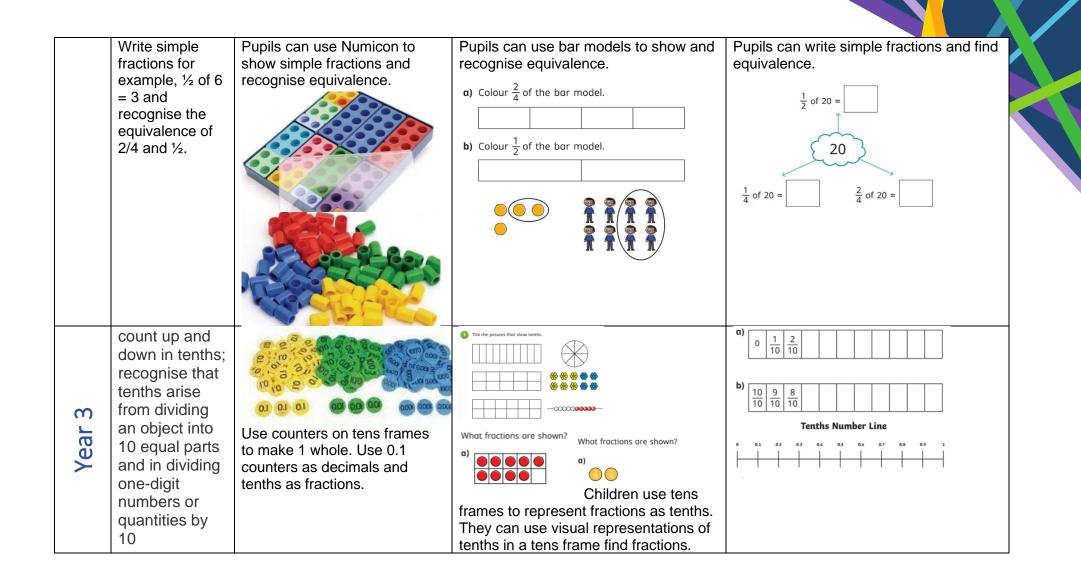
remainder in the	1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
tens.	2 2) <u>5</u> 8	$2^{+}$	$\begin{array}{r} t \circ \\ 29 \\ 2 \end{array}$
	Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens but there is a remainder!	To find it, multiply $2 \times 2 = 4$ , write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.
	1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
	t o 2 <mark>9</mark> 2 ) 5 8 <u>- 4</u> 1 8	t o 2 9 2 ) 5 8 - 4 - 1 8 0	2 ) 5 8 - 4 - 1 8 - 1 8 0
	Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.
	L	1	



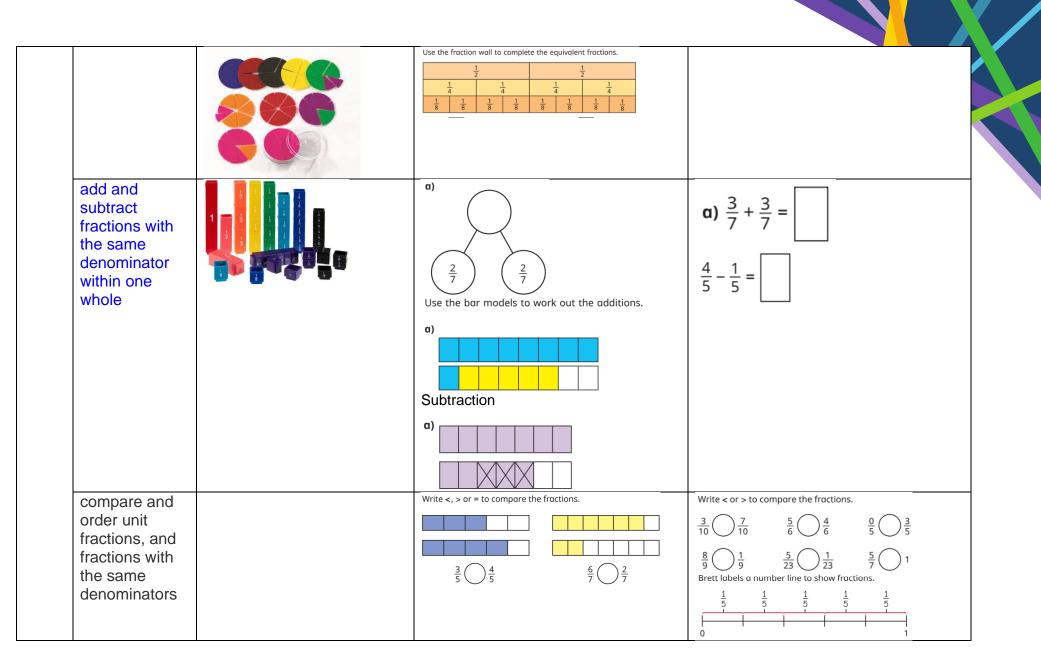
The following sections are ideas of how the CPA can be used to support learners within different national curriculum objectives. The list of ideas is not exhaustive.

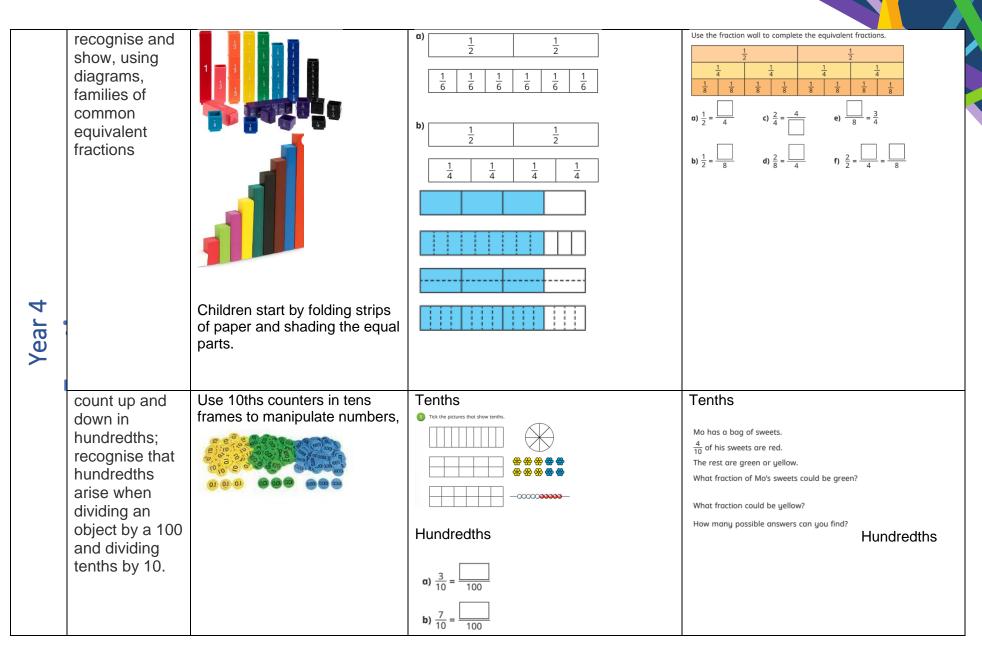
Objective, strategy and key	Concrete	Pictorial	Abstract
Vocabulary.         Recognise, find and name a half as one of two equal parts of an object, shape or quantity.	Pupils can use fraction pizzas/shapes to recognise a half.	Pupils can recognise a half of a shape, objects and quantities by drawing them or seeing them drawn with potential lines drawn on.         Image: Constraint of the series of the serie	Pupils can find a half of a quantity and are secure in doing this using concrete and pictorial resources.

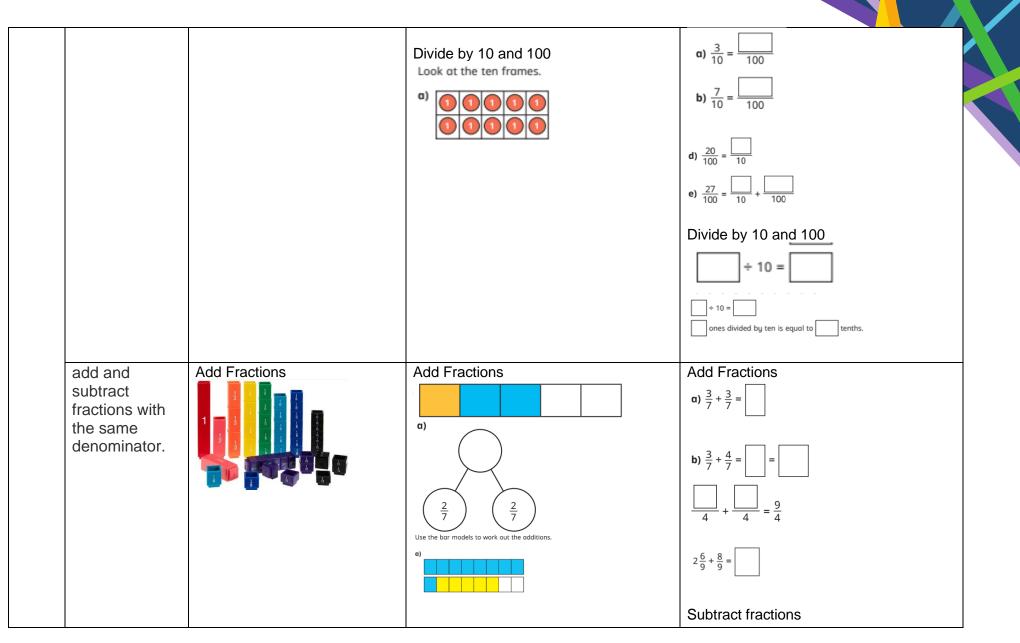
	Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.	Pupils can use counters to recognise/find a quarter.         Use the counters to complete the statement.         One quarter of 16 is         Use counters to complete the sentences.         A quarter of 4 is         1 is one quarter of	Pupils can use pictures drawn or pre- drawn to recognise/find a quarter of objects and shapes.	<pre>Once secure, pupils can find a quarter of objects and amounts.  Sure each quantity into four equal groups.  There arecakes. There iscake in each quarter. A quarter ofis  There arepeaches. There are _</pre>
Year 2	Recognise, find, name and write fractions 1/3, ¼, 2/4 and 3/4 of a length, shape, set of objects or quantity.	Pupils can use fraction tiles, fraction pizzas to recognise a 1/3, ¼, 2/4 and 3/4.	Pupils can recognise/find 1/3, ¼, 2/4 and ¾ from their drawings or from pictures.	Pupils can find 1/3. ¼, 2/4 and ¾ of amounts and present this in words/numbers.

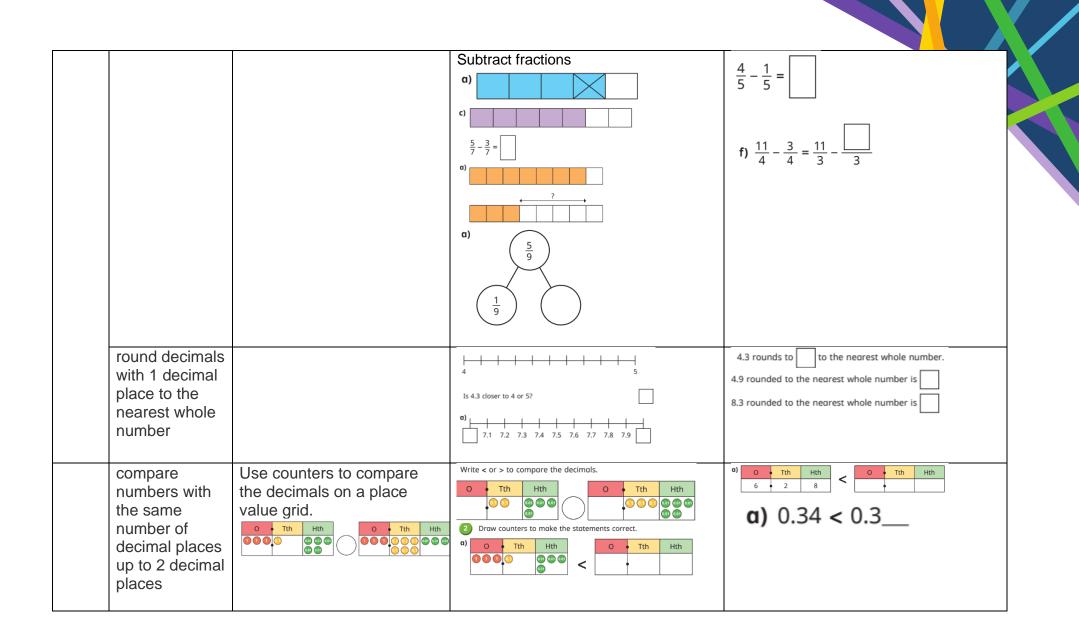


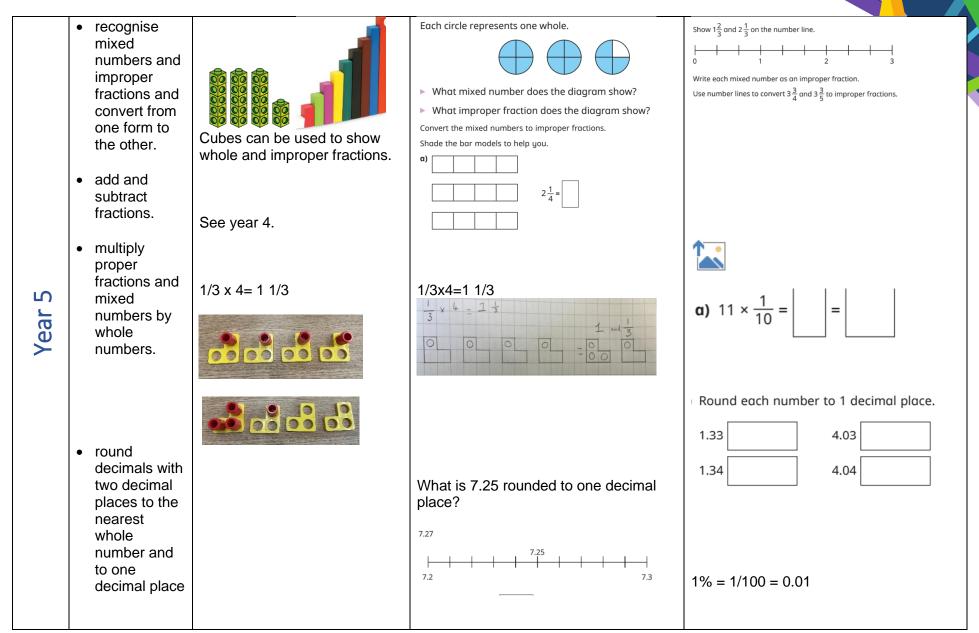
recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators	Shade $\frac{2}{5}$ of each shape.         Image: the shape shade shape shade shape shade shape	
recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominator	Count forwards to complete the number lines. 6 $1$ $2$ $5$ $5$ $5$	Estimate where the fractions belong on the number line. $1 \\ 3 \\ 4 \\ 1 \\ 2 \\ 8 \\ 9 \\ 1 \\ 1 \\ 3 \\ 1 \\ 1 \\ 3 \\ 1 \\ 1 \\ 1 \\ 1$
recognise and show, using diagrams, equivalent fractions with small denominators	Shade the diagrams to help you complete the equivalent fractions. The first one has been done for you. (a) (b) (c) (c) (c) (c) (c) (c) (c) (c) (c) (c	Use the double number line to complete the equivalent fractions. $ \begin{array}{ccccccccccccccccccccccccccccccccccc$

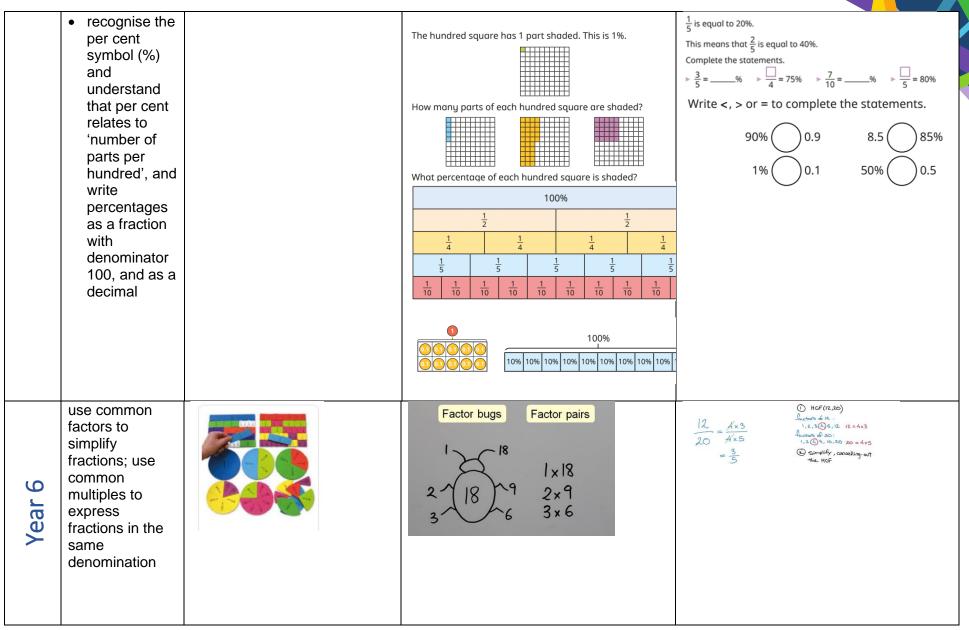




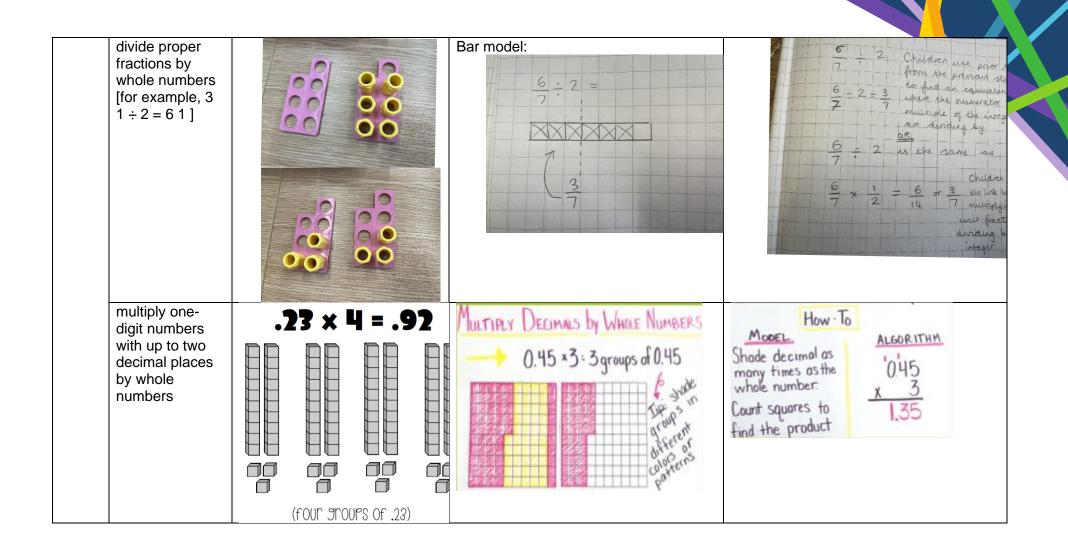








add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions		13 13 13 13 14 14 14 14 14 14 14 14 14 14	How to Add Fractions Some Denominators $\frac{1}{5} + \frac{3}{5}$ $\frac{1+3}{5} = (\frac{4}{5})$ $\frac{7+6}{14} = (\frac{13}{14})$
multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 4 $1 \times 21 = 81$ ]	See Year 5	See Year 5	See Year 5



	Measure						
Objective, strategy	Concrete	Pictorial	Abstract				
and key							
vocabulary.							
Compare, describe and solve <b>practical</b> problems for: • lengths and heights [for example, long/short, longer/shorter, tall/short, double/half] • mass/weight [for example, heavy/light, heavier than, lighter than] • capacity and volume [for example, full/empty, more than, less than, half, half full, quarter] • time [for example, quicker,	Pupils can use a <u>variety of</u> <u>measuring equipment</u> , scales, cups/jugs and clocks/stop watches to describe and compare measurements.	Use representations like below:	Pupils can use the vocabulary taught to compare and describe measurements. Write longer or shorter to compare the ribbons. The plain ribbon is than the stripy ribbon. The stripy ribbon is than the plain ribbon. Write heavier or lighter to complete the sentence. The bottle is than the can.				

Year 2	<ul> <li>recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value.</li> <li>find different combinations of coins that equal the same amounts of money.</li> </ul>		Draw coins/notes.	
	• tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times.	Use clocks.	Draw clocks/use clock stamps or blank worksheets.	

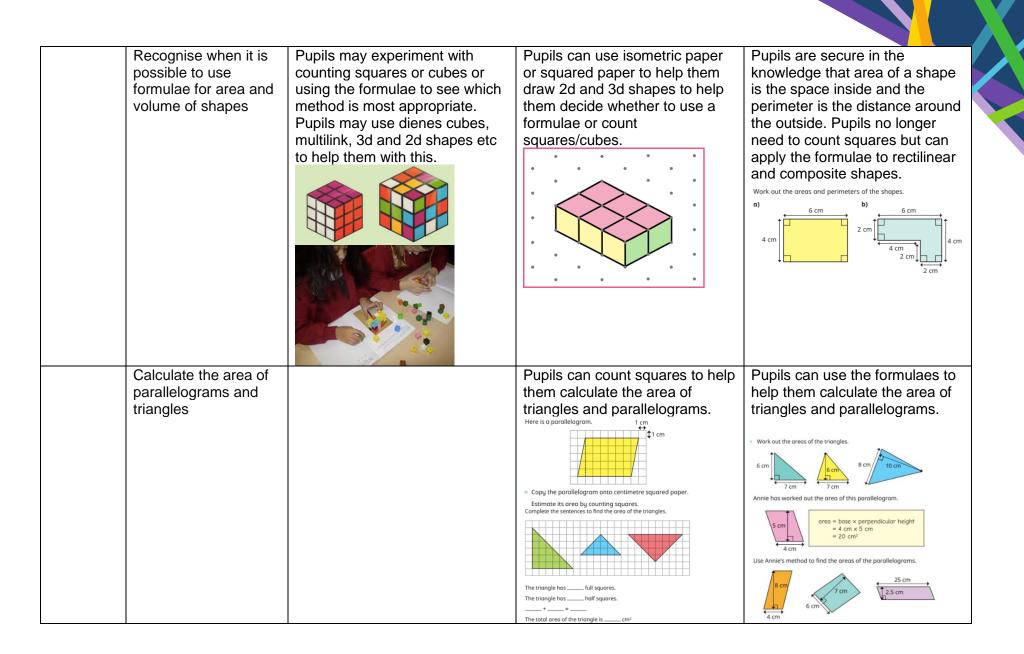
	<u>.</u>		
measure, compare,	Children to use and handle a		If the mass of two apples is 50g,
add and subtract:	range of measure resources to	Complete the bar models.	what is the mass of one apple?
lengths (m/cm/mm);	aid learning.	? 5 kg ? 500 g	
<b>e</b>		6 kg and 900 g 3 kg and 900 g	Complete the sentence for each arrow.
		3 kg and 100 g 1 kg and 450 g	$kg 0 \xrightarrow{B} A \xrightarrow{C} 1$
		2	g 0 1,000
(//111)	the first the first	·	Arrow is pointing to g.
	A A A A A A A A A A A A A A A A A A A		What fraction of a kilogram is each arrow pointing to?
			Complete the number sentences.
			<ul> <li>&gt; 30 ml + 70 ml = ml</li> <li>&gt; 300 ml + 700 ml = ml</li> <li>&gt; 450 ml + 55 ml = ml</li> <li>&gt; 450 ml + 550 ml = ml</li> </ul>
			▶ 100 ml - 38 ml = ml ▶ 1,000 ml - 380 ml = ml
			21 ml + ml = 100 ml 210 ml + ml = 1,000 ml
	000		ml + 340 ml = 1,000 ml
	• •		
add and subtract			
amounts of money	to aid learning.		
to give change,			
using both £ and p	As Year 2		
tell and write the	To use <b>analogue clocks</b> to		
	•		
0			
estimate and read			
time with increasing			
	add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml) add and subtract amounts of money to give change, using both £ and p in practical contexts tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12- hour and 24-hour clocks estimate and read	add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (I/mI)range of measure resources to aid learning.add and subtract amounts of money to give change, using both £ and p in practical contextsUse physical coins and notes to aid learning.add and write the time from an analogue clock, including using Roman numerals from I to XII, and 12- hour and 24-hour clocks estimate and readUse analogue clocks to manipulate numerals and roman numerals.	add and subtract:       range of measure resources to aid learning.         iengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)       if learning.         add and subtract amounts of money to give change, using both £ and p in practical contexts       Use <u>physical coins</u> and <u>notes</u> to aid learning.         add and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour clocks       To use <u>analogue clocks</u> to manipulate numerals and roman numerals. As Year 2

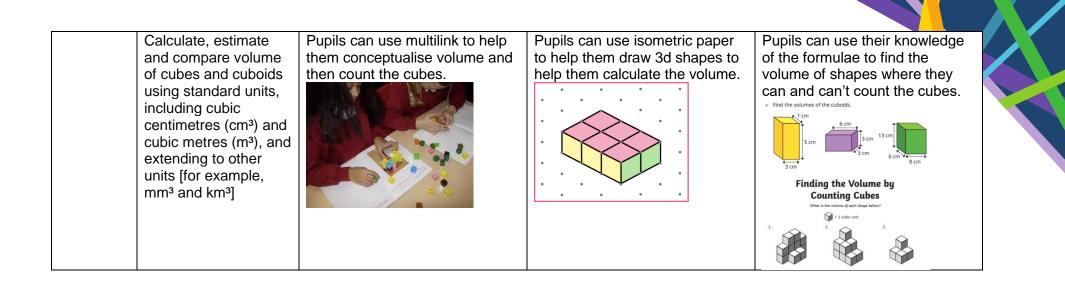
	accuracy to the nearest minute.			
	measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres.	Use squares to create the shape.	Draw the shape in books using the squares.         1       2       3       4       5       6         18       -       -       7       7         17       -       -       8       9         16       -       -       9         15       14       13       12       11       10	
Year 4	find the area of rectilinear shapes by counting squares.	Situation Cris	Draw the shape in books using the squares.         1       2       3       4       5       6         7       8       9       10       11       12         13       14       15       16       17       18	

		measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres	As year 4.			
Year 5		calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm2) and square metres (m2) and estimate the area of irregular shapes	As year 4.			
		estimate volume [for example, using 1 cm3 blocks to build cuboids (including cubes)] and capacity [for example, using water]	Use building blocks.	Use dotted paper to support.	L x W x H =	
Year 6	Equal to, approximately equal	Solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 decimal places where appropriate.	Pupils can use PV charts and PV counters to help them with x and dividing by 10, 100 and 1000 in converting units of measure. Pupils will need the conversion charts to help remind them.	Pupils exposed to tables, bar models, conversion charts and PV charts help them convert between different units of measure. Complete the diagram to show the conversions.	Pupils are secure in multiplying and dividing by 10, 100 and 1000 and can do this without using the PV chart to help them convert.	

Use, read, write and	Pupils should be able to handle	Pupils use tables, bar models,	Pupils can read, write and
convert between	and have access to concrete	conversion charts and PV charts	convert between units of
standard units,	examples of items which show	help them convert between	measure.
converting	units of measure e.g. rulers of	different units of measure.	Write <, > or = to compare the measurements. 100 mt $(0.1 \ l \ 15 \ cm \ 1.5 \ m$
measurements of	various sizes, measure tapes,	Complete the diagram to show the conversions.	
length, mass, volume	measuring scales and weights,	÷ ÷100 ÷	25 L 2,500 ml 1,500 mm 1, $\frac{1}{2}$ m
and time from a	containers etc.		4,020 ml 4.2 l 1.5 km 150 m
smaller unit of	US AND METRIC SYSTEMS	mm cm m km	A bag of flour has a mass of 200 g.
measure to a larger		× × ×	Scott uses 3 bags of flour when baking.
unit, and vice versa,	165		How much flour does he use? Flour Write your answer in kilograms.
using decimal notation			
to up to 3 decimal			
places	© Study.com		
	Pupils can use PV charts and		
	PV counters to help them with x		
	and dividing by 10, 100 and 1000 in converting units of		
	measure. Pupils will need the		
	conversion charts to help remind		
	them.		
Convert between	Pupils could make groups of 5	Pupils can use bar models and	Pupils recall the conversions
miles and kilometres	and groups of 8 to help them	number lines to help them	between miles and kilometres
	convert between miles and	understand the relationship	and use these to convert – need
	kilometres using the fact that 5	between miles and kilometres	to be secure in multiplying and
	miles is approximately equal to 8		dividing.
	kilometres.		
			Complete the conversions.
			7.5 miles ~ km
			▶ 160 km ~ miles ▶ miles ~ 320 km
			▶ 96 miles ~ km   ▶ km ~ 250 miles

		Fill in the missing numbers on the number line.	
	Muttpiloation Table         Image: Constraint of the constraint	kilometres 0 1.6 2.5 miles 8 kilometres 5 miles 8 kilometres 5 miles 8 kilometres 5 miles 16 km is miles 16 km is miles	
	<ul> <li>2 km away from your school</li> </ul>		
Recognise that shapes with the same areas can have different perimeters and vice versa.	Pupils may use equipment to calculate area and perimeter of different shapes in order to identify the differences.	Pupils can look at pictorial drawings of shapes to calculate area and perimeter and compare the appearance of the shapes. Which two rectangles have the same area? Which two rectangles have the same area? Mage defined a same area? Ma	Pupils use their knowledge of factor pairs to help them calculate the area of shapes and then draw them to understand that they may look different. Draw as many rectangles as possible that have these areas. All the side lengths should be whole numbers. ▶ 36 cm <sup>2</sup> ▶ 16 cm <sup>2</sup> ▶ 17 cm <sup>2</sup>





		ΔΙ	aobro			
	Algebra Input, Output, Function, Rule, Inverse Operations					
	Objective, strategy and key vocabulary.	Concrete	Pictorial	Abstract		
Year 6	Use simple formulae	Represent each person's age using cuisenaire rods, place each person above each other to show comparisons. Andy Bella Colin Co	Represent each person's age using cuisenaire rods, place each person above each other to show comparisons.	Students should represent each person's age as an algebraic expression. A X B X+2 C X-1 D 2X+4 This could be represented as $2(x+2)$ depending on student's ability 3x - 1 = 5 (+1) 3x = 2 (+2) 3x = 2		
	Generate and describe linear number sequences	It will be beneficial if students use the manipulatives on top of a whiteboard for this stage. Students should identify that the sequence increases by 3 each time. 4, 7, 10, 13, 4, 7, 10, 13, Students should then know that the sequence is linked to the 3 times table. Students should then represent this using the manipulative. Since the sequence increases by 3, the 3 block will be needed.	Term (a) 1 $\sqrt[3]{3}/\sqrt[3]{1}$ $3 \times 1 + 1 = 4$ 2 $\sqrt[3]{3}/\sqrt[3]{3}/\sqrt[3]{1}$ $3 \times 2 + 1 = 7$ 3 $\sqrt[3]{3}/\sqrt[3]{3}/\sqrt[3]{3}/\sqrt[3]{1}$ $3 \times 3 \neq 1 = 70$ $0$ $3 \times 1 = 70$ $0$ $3 \times 1 = 10$ $0$ $3 \times 1 + 1 \times 3 + 1$ It may be useful for weaker students to shade in the term-to-term blocks to make them easier to count and distinguish.	Students however MUST be able to clearly justify why by explaining that is the sequence is "one more than the three times table, therefore the rule is 3n+1."		
	Express missing number problems algebraically	Create a representation to show the equation $2m + 2 = 6$ Pupils could begin by handling a range of physical objects and	Bar models, part-whole models and pictorial drawings of concrete objects	Pupils can expressing missing number problems using numbers and symbols, as well		

	then using representations to show expressions and then create and solve equations.	b 3 3 3 3 3 83 60 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	as being able to solve them from word problems. The total cost of a scarf and a book is £17 Form an equation to represent this information.
Find pairs of numbers that satisfy an equation with 2 unknowns.	Pairs of values are represented using 2D shapes or other concrete resources e.g. circle + square = 5. What could the pairs of values be?	Pupils can then start using strategies such as a table to efficiently find the pairs of numbers that satisfy an equation. They could also draw their pairs of numbers using symbols or pictures.	Pairs of values are represented in abstract form using numbers. Missing numbers are represented using algebraic $\boxed{x*y=12}$ Find six possible pairs of values for x and y. $\boxed{x}$ symbols.
Enumerate possibilities of combinations of 2 variables	Possibilities are represented by pupils using 2D shapes and other concrete resources, same as in the previous objective.	Pupils continue to use a table to efficiently find possibilities of combinations of different	Pupils start to enumerate possibilities mentally e.g. by using knowledge of number bonds, times tables etc. They can then apply this objective to real-life problems such as finding the lengths of shapes –

		(3) + (4) + (3) = 22 $(4) + (4) + (3) = 28$ $(4) + (4) + (3) = 28$ $(4) + (4) + (3) = 38$ $(4) + (4) + (3) = 38$ $(4) + (4) + (3) = 38$ $(4) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$ $(5) + (4) + (3) = 38$	Pumbers. a and b are whole numbers: $ \begin{array}{c}         2a + b = 1a \\         1a + 1a \\ $	necessary for calculating area and perimeter. x ord y ore both multiples of 5 less than 100 If $2x = y$ , crite the possible values of x ord y. x = 20, y = 10 x = 20, y = 10 x = 30, y = 20 x = 30, y = 20 x = 30, y = 20 x = 50, y = 20 x = 0, y = 10 x = 0, y = 0 x = 0, y = 0, y = 0, y = 0 x = 0, y = 0, y = 0, y = 0 x = 0, y = 0, y = 0, y = 0 x = 0, y = 0, y = 0, y = 0 x = 0, y = 0, y = 0, y = 0, y = 0 x = 0, y = 0, y = 0, y = 0, y = 0 x = 0, y	
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